通量频率损失修正：能量谱，协谱，传输方程（频率损失方程）及频率修正方程

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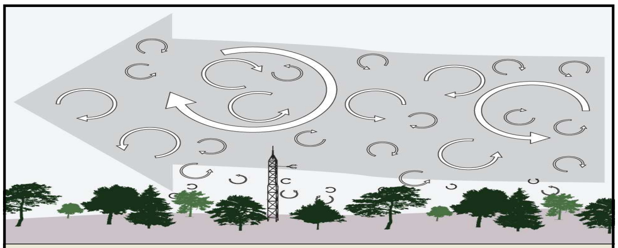
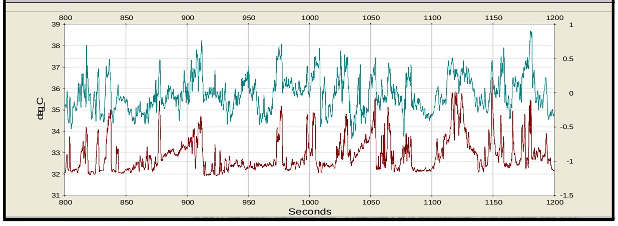
Campbell Scientific, US

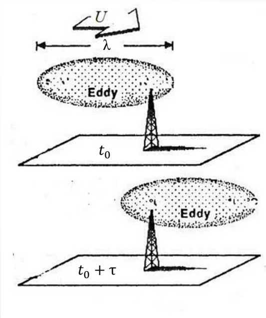
第14次 ChinaFLUX 通量理论与技术培训

2019年8月7日





湍流团的大小与频率



$$周期\ (\tau) = \frac{波长\ (\lambda)}{风速\ (u)} = \frac{1}{频率\ (f)}$$

垂直风速的湍流谱

ω — 角频率

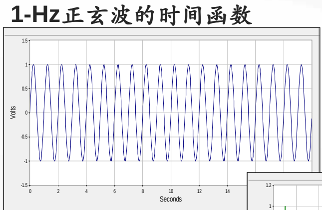
t — 时间

$$w(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} w(t)\, e^{i\omega\tau} d\tau$$

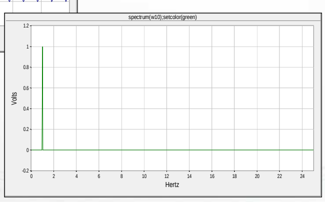
$$w(t) = \int_{-\infty}^{\infty} w(\omega) e^{-i\omega t} d\omega$$

正玄波从时域到频的转换

1-Hz正玄波的时间函数

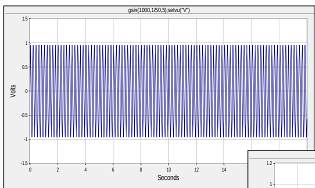


1-Hz正玄波的频率函数

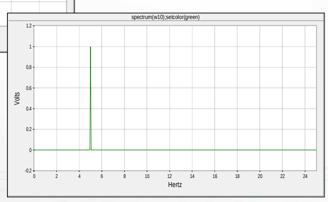


正玄波从时域到频的转换

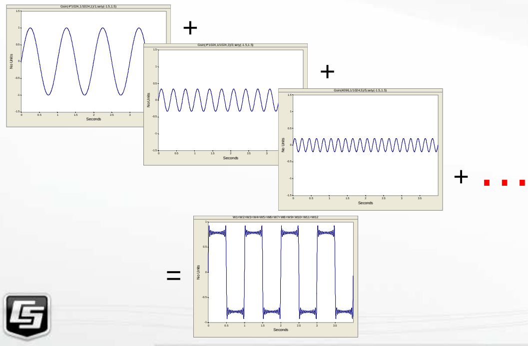
5-Hz正玄波的时间函数



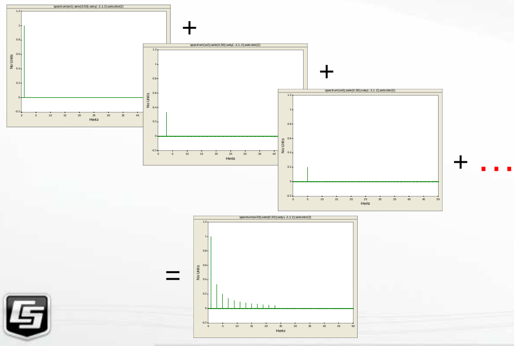
5-Hz正玄波的频率函数



正玄波时域函数叠加



正玄波频域函数叠加



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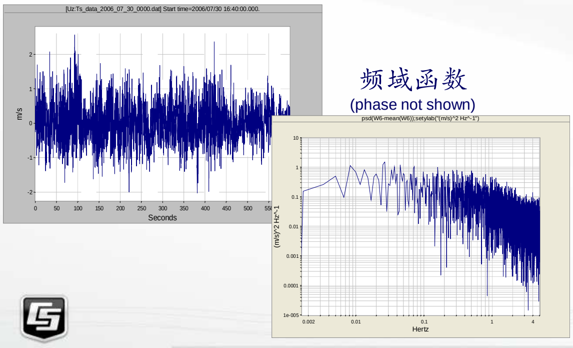
垂直风速

时域函数



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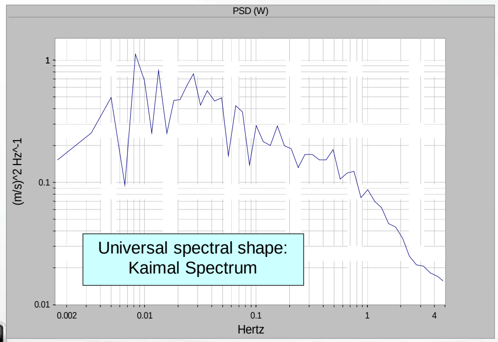
平滑垂直风速能量谱密度



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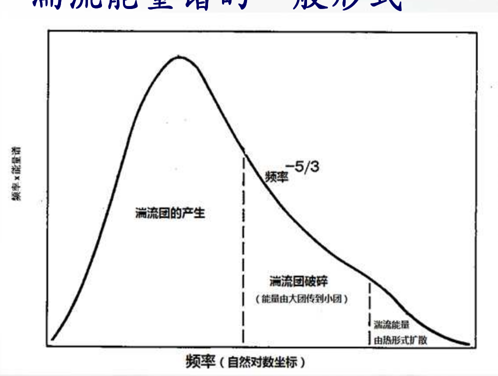
湍流能量谱的一般形式



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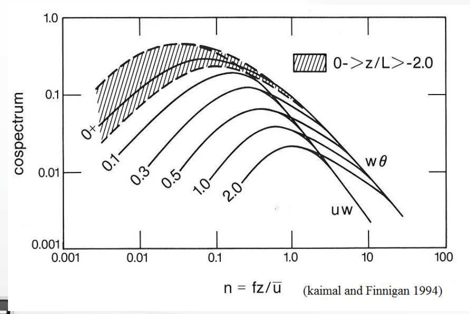
垂直风速与水平风速/温度在
不同边界层稳定条件下的湍流协谱



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方差是对能量谱密度函数在频域上的积分

自相关定义

$$R_{ww}(\tau) = \overline{w'(t)w'(t+\tau)} = \lim_{T_p\to\infty}\frac{1}{2T_p}\int_{-T_p}^{T_p} w'(t)w'(t+\tau)d\tau$$

傅立叶变换对

$$S_{ww}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{ww}(\tau)e^{i\omega\tau}d\tau$$

$$R_{ww}(0) = \int_{-\infty}^{\infty} S_{ww}(\omega)e^{-i\omega 0}d\omega = \int_{-\infty}^{\infty} S_{ww}(\omega)d\omega = \overline{w'(t)w'(t+0)} = \overline{w'^2}$$

$$\overline{w'^2} = \int_{-\infty}^{\infty} S_{ww}(\omega)d\omega$$

协方差是对协谱密度函数在频域上的积分

自相关定义

$$R_{\alpha w}(\tau) = \overline{\alpha'(t)w'(t+\tau)} = \lim_{T_p\to\infty}\frac{1}{2T_p}\int_{-T_p}^{T_p} w'(t)w'(t+\tau)d\tau$$

傅立叶变换对

$$S_{\alpha w}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{\alpha w}(\tau)e^{i\omega\tau}d\tau$$

$$R_{\alpha w}(0) = \int_{-\infty}^{\infty} S_{\alpha w}(\omega)e^{-i\omega 0}d\omega = \int_{-\infty}^{\infty} S_{\alpha w}(\omega)d\omega = \overline{\alpha'(t)w'(t+0)} = \overline{\alpha' w'}$$

$$\overline{\alpha' w'} = \int_{-\infty}^{\infty} S_{\alpha w}(\omega)d\omega$$

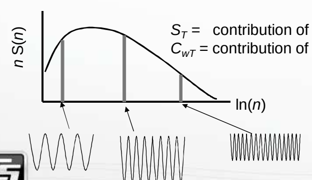
湍流谱和协湍流谱的一般形状

$$\text{variance} = \overline{T'^2} = \int_0^\infty nS_T(n)\,d\ln(n)$$

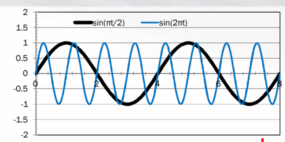
$$\text{covariance} = \overline{wT'} = \int_0^\infty nC_{wT}(n)\,d\ln(n)$$

(Co)variance is area under the (co)spectral density curve

S_T = contribution of total variance per unit *dn*
C_{wT} = contribution of total covariance per unit *dn*



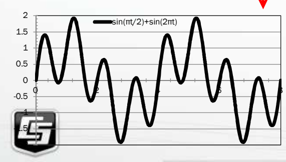
Larry Jacobsen, CSI

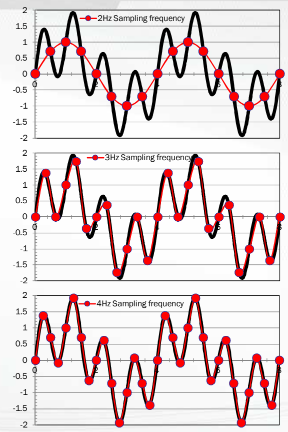


测定频率与湍流信号频率

香农定理(Shannon Theorem)

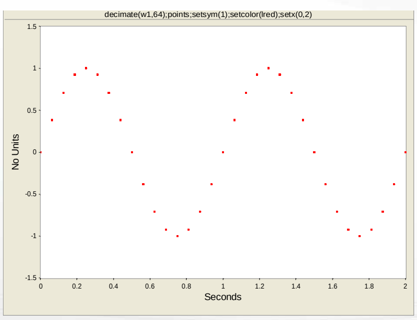
测定频率最低应是被测信号最高频率的二倍。



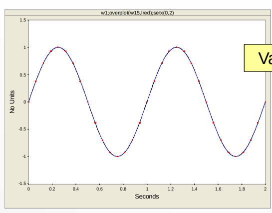


Aliasing

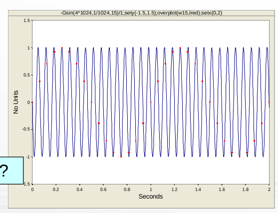
离散型测得的信号



Aliased 信号的统计描述



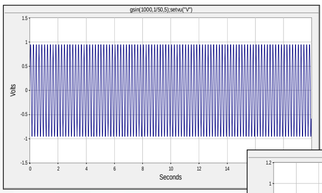
Variance = ???



Variance = ???

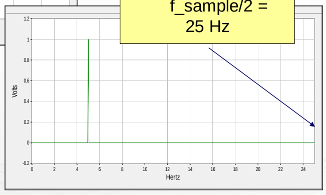
测得信号的Nyquist 频率 （正玄波）

5-Hz 正玄波的时间函数 （测定频率为 50 Hz）



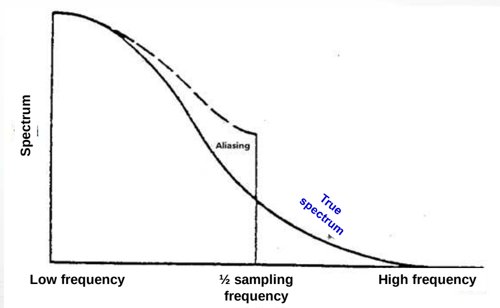
5-Hz 正玄波的频率函数，（测定频率为 50 Hz）

Nyquist Frequency = f_sample/2 = 25 Hz



Aliasing:

1. 谱在谱线上，大于1/2采样频率的信号不显示在它自己频率上。
2. 然而部分地显示在小于Nyquist 频率范围内。



数学理解 aliasing

$$w(t) = \sin 2\pi\left(\frac{p+0.25}{2\times 0.005}\right)t$$

If $p = 1$, Then $\frac{1+0.25}{2\times 0.005} = 125$

200 Hz 采样频率 (周期 = 0.005 sec)

$$w(t_i) = \sin 2\pi\left(\frac{p+0.25}{2\times 0.005}\right)(t_i \times 0.005) = \cos p\pi t_i \sin 0.25\pi t_i$$

整数

p 为奇数 (p=1，125 Hz 信号)

$$w(t_i) = (-1)^{t_i} \sin 0.25\pi t_i$$
$$= \cos \pi t_i \sin 0.25\pi t_i$$
$$= -\sin(2\pi)\times 75\times(0.005t_i)$$

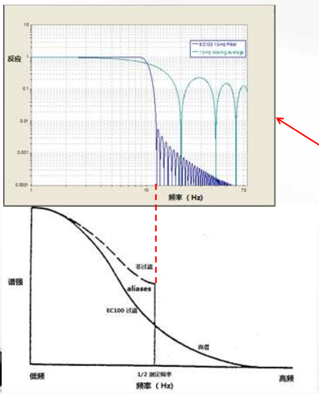
200Hz

p 为偶数 (p=2)，225 Hz 信号

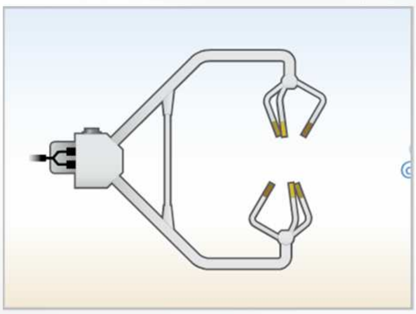
$$w(t_i) = \sin 0.25\pi t_i)$$
$$= \sin(2\pi)\times 25\times(0.005t_i)$$

200Hz

减少低Aliasing效应

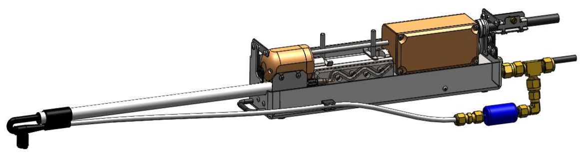


传感器结构与高频湍流图



线形路径平均

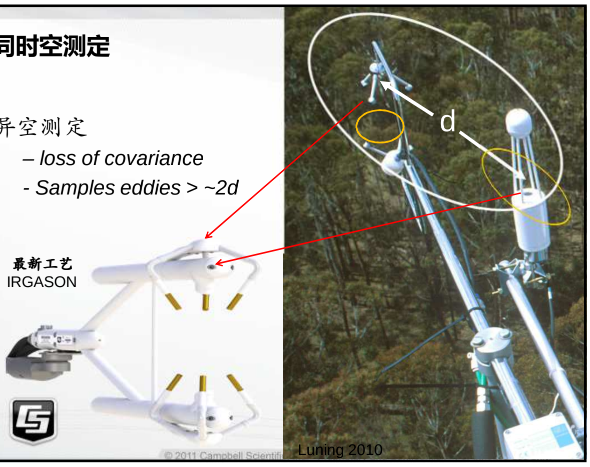
闭路系统频率损失



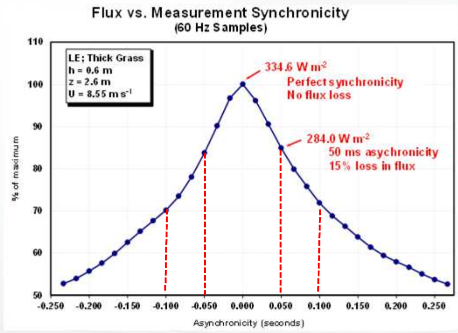
同时空测定

异空测定

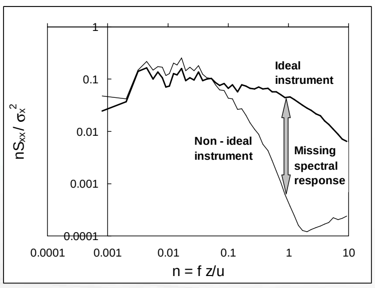
- – *loss of covariance*
- - *Samples eddies > ~2d*

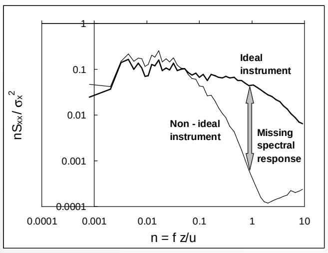


同时测定



高频通量损失的湍流谱





$$T_{\alpha w}(n) = \frac{S_{\alpha w\_m}(n)}{S_{\alpha w}(n)}$$

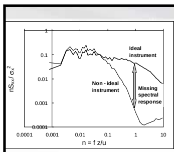
$$S_{\alpha w\_m}(n) = T_{\beta w}(n) S_{\alpha w}(n)$$

$$S_{\alpha w\_m}(n) = T_{\alpha w}(n) S_{\alpha w}(n)$$

$$\int_0^\infty nS_{\alpha w\_m}(n) d\ln n = \int_0^\infty nT_{\alpha w}(n) S_{\alpha w}(n) d\ln n$$

$$\frac{1}{\int_0^\infty nS_{\alpha w\_m}(n) d\ln n} = \frac{1}{\int_0^\infty nT_{\alpha w}(n) S_{\alpha w}(n) d\ln n}$$

$$\frac{\int_0^\infty nS_{\alpha w}(n) d\ln n}{\int_0^\infty nS_{\alpha w\_m}(n) d\ln n} = \frac{\int_0^\infty nS_{\alpha w}(n) d\ln n}{\int_0^\infty nT_{\alpha w}(n) S_{\alpha w}(n) d\ln n}$$



$$\frac{\int_0^\infty nS_{\alpha w}(n) d\ln n}{\int_0^\infty nS_{\alpha w\_m}(n) d\ln n} = \frac{\int_0^\infty nS_{\alpha w}(n) d\ln n}{\int_0^\infty nT_{\alpha w}(n) dS_{\alpha w}(n) d\ln n}$$

$$\text{variance} = \overline{T'^2} = \int_0^\infty nS_T(n)\, d\ln(n)$$

$$\text{covariance} = \overline{w'T'} = \int_0^\infty nC_{wT}(n)\, d\ln(n)$$

$$\overline{\alpha' w'} = \int_0^\infty nS_{\alpha w}(n) d\ln n$$

$$\left(\overline{\alpha' w'}\right)_m = \int_0^\infty nS_{\alpha w\_m}(n) d\ln n$$

$$\frac{\overline{\alpha' w'}}{\left(\overline{\alpha' w'}\right)_m} = \frac{\int_0^\infty nS_{\alpha w}(n) d\ln n}{\int_0^\infty nT_{\alpha w}(n) S_{\alpha w}(n) d\ln n}$$

$$\frac{\overline{\alpha' w'}}{\left(\overline{\alpha' w'}\right)_m} = \frac{\int_0^\infty n\frac{S_{\alpha w}(n)}{\overline{\alpha' w'}} d\ln n}{\int_0^\infty nT_{\beta w}(n)\frac{S_{\alpha w}(n)}{\overline{\alpha' w'}} \ln n}$$

$$\overline{\alpha' w'} = \left(\overline{\alpha' w'}\right)_m \frac{\int_0^\infty nS_{\alpha w\_model}(n) d\ln n}{\int_0^\infty nT_{\alpha w}(n) S_{\alpha w\_model}(n) d\ln n}$$

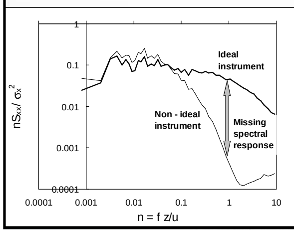
频率修正

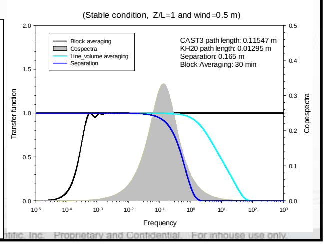
$$\overline{\alpha' w'} = \left(\overline{\alpha' w'}\right)_m \left\{ \frac{\int_0^\infty nS_{\alpha w\_model}(n) d\ln n}{\int_0^\infty nT_{\alpha w}(n) S_{\alpha w\_model}(n) d\ln} \right\}$$

'真' 协谱

测得的协方差

频率反应函数





频率修正系数

$$C_f = \frac{\int_0^\infty nS_{\alpha w\_model}(n) d\ln n}{\int_0^\infty nT_{\beta w}(n) S_{\alpha w\_model}(n) d\ln n} \geq 1$$

$$\overline{\alpha' w'} = C_f \left(\overline{\alpha' w'}\right)_m$$

$$S_{\alpha w\_model}$$

历史与应用

协湍流谱

$S_{\alpha w\_model}$

$\frac{Z}{L} > 0$

垂直风速与水平风速

$$fS_{uw}(f) = \frac{fz/\bar{u}}{A_{uw} + B_{uw}\left(\frac{fz}{\bar{u}}\right)^{2.1}}$$

$$A_{uw} = 0.124\left(1+7.9\frac{z}{L}\right)^{0.75}$$

$$B_{uw} = 23.252\left(1+7.9\frac{z}{L}\right)^{-0.825}$$

垂直风速与气温

$$fS_{Tw}(f) = \frac{fz/\bar{u}}{A_{Tw} + B_{Tw}\left(\frac{fz}{\bar{u}}\right)^{2.1}}$$

$$A_{Tw} = 0.284\left(1+6.4\frac{z}{L}\right)^{0.75}$$

$$B_{Tw} = 9.3447\left(1+6.4\frac{z}{L}\right)^{-0.825}$$

协湍流谱

$S_{\alpha w\_model}$

$\frac{Z}{L} =< 0$

垂直风速与水平风速

$$fS_{uw}(f) = \begin{cases} \dfrac{20.78\ fz/u}{\left(1+\dfrac{31z}{u}f\right)^{1.575}} & \dfrac{z}{u}f < 0.24 \\ \dfrac{12.66\ fz/u}{\left(1+\dfrac{9.6z}{u}f\right)^{2.4}} & \dfrac{z}{u}f \geq 0.24 \end{cases}$$

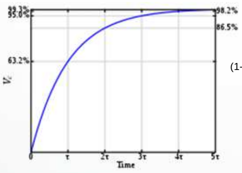
垂直风速与气温

$$fS_{Tw}(f) = \begin{cases} \dfrac{12.92\ fz/u}{\left(1+\dfrac{26.7z}{u}f\right)^{1.375}} & \dfrac{z}{u}f < 0.54 \\ \dfrac{4.378\ fz/u}{\left(1+\dfrac{3.8z}{u}f\right)^{2.4}} & \dfrac{z}{u}f \geq 0.54 \end{cases}$$

$T_{\alpha w}$

研究与评估仪器

传感器反应速度与高频湍流团



时间常数(τ_{FW})

传感器感应到63.2%被测物理量所需的时间。

$(1-\frac{1}{e})=0.632$

$$\tau_{FW} = 0.167D^2\frac{\rho_{FW}C_{FW}}{k_a\mathrm{Nu}}$$

D dimension
ρ_{FW} material density of thermocouple
C_{FW} specific heat of thermocouple materials
k_a thermal conductivity of air
Nu Nusselt number

时间常数（τ）的理论定义

$$\frac{dT_m(t)}{dt} = \frac{T(t)-T_m(t)}{\tau}$$

$T(t)$ 时间t时的被测温度

$T_m(t)$ 时间t时传感器测得的温度

频率反应函数

对时间常数定义方程两端做傅立叶变换

$$\mathcal{F}\left(\frac{dT_m(t)}{dt}\right) = \mathcal{F}\left(\frac{T(t)-T_m(t)}{\tau}\right)$$

即

$$-j\omega T_m(\omega) = \frac{1}{\tau}[T(\omega)-T_m(\omega)]$$

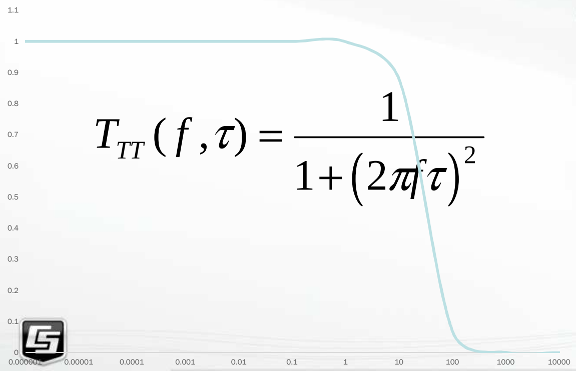
ω - 频率， $j=\sqrt{-1}$.

整理得

$$\frac{T_m(\omega)}{T(\omega)} = \frac{1}{1-j\tau\omega}$$

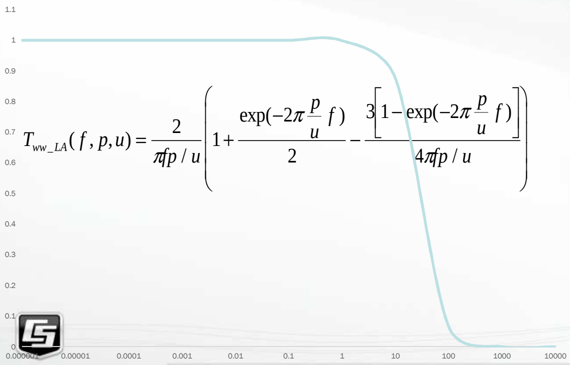
$$T_T(f)=\frac{T_m(f)}{T(f)}=\frac{1}{1-j2\pi f\tau}$$

$$T_T(f)=\frac{1}{\sqrt{1+(2\pi f\tau)^2}}$$



$$T_{TT}(f,\tau)=\frac{1}{1+(2\pi f\tau)^2}$$

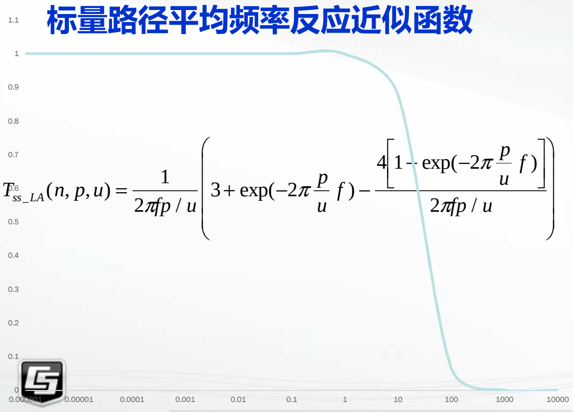
垂直风速路径平均频率反应近似函数



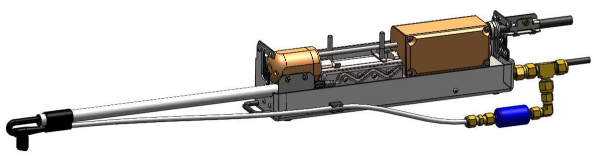
$$T_{ww\_LA}(f,p,u)=\frac{2}{\pi f p/u}\left(1+\frac{\exp(-2\pi\frac{p}{u}f)}{2}-\frac{3\left[1-\exp(-2\pi\frac{p}{u}f)\right]}{4\pi f p/u}\right)$$

$$T_{Tw}(f)=T_T(f)T_{w\_LA}(f)$$

标量路径平均频率反应近似函数

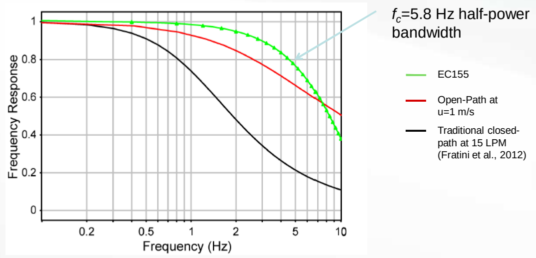


$$T_{ss\_LA}(n,p,u)=\frac{1}{2\pi f p/u}\left(3+\exp(-2\pi\frac{p}{u}f)-\frac{4\left[1-\exp(-2\pi\frac{p}{u}f)\right]}{2\pi f p/u}\right)$$

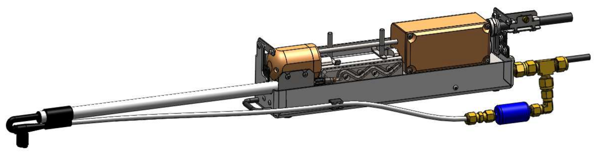


$$T_{CO2}(f)=\frac{1}{\sqrt{1+\left(2\pi\frac{f}{f_{c\_CO2}}\right)^2}}$$

闭路涡动系统的频率反应

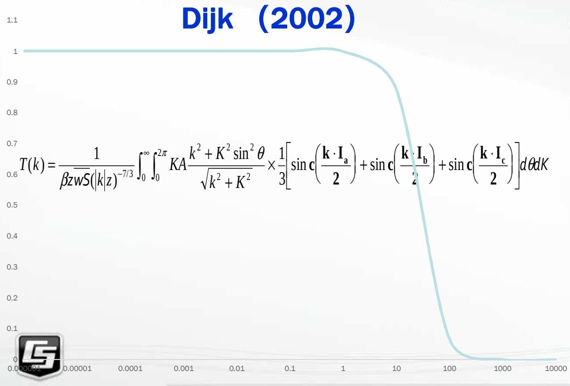


- Note under higher wind conditions, the open-path frequency response will be better than a closed-path.
- Closed-path frequency response does not change with wind speed.
- The open-path frequency response in this model is limited by path averaging.

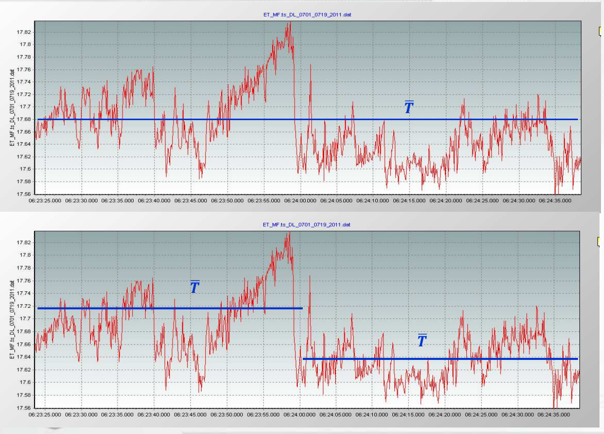


$$T_{CO2}(f)=\frac{1}{\sqrt{1+\left(2\pi\frac{f}{4.6}\right)^2}}$$

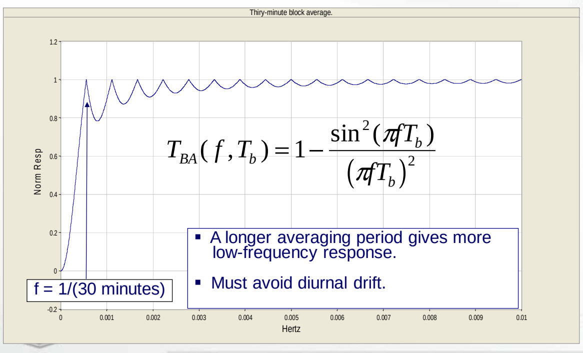
CSAT超声热通量路径平均频率反应函数 Dijk （2002）



$$T(k)=\frac{1}{\beta z w S(|k|z)^{-7/3}}\int_0^\infty\int_0^{2\pi} KA\frac{k^2+K^2\sin^2\theta}{\sqrt{k^2+K^2}}\times\frac{1}{3}\left[\mathrm{sinc}\left(\frac{\mathbf{k}\cdot\mathbf{l}_a}{2}\right)+\mathrm{sinc}\left(\frac{\mathbf{k}\cdot\mathbf{l}_b}{2}\right)+\mathrm{sinc}\left(\frac{\mathbf{k}\cdot\mathbf{l}_c}{2}\right)\right]d\theta\, dK$$



平均方法 (平均时长)



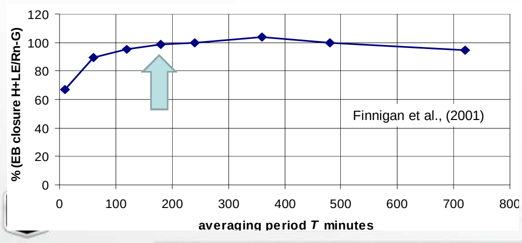
$$T_{BA}(f,T_b)=1-\frac{\sin^2(\pi f T_b)}{(\pi f T_b)^2}$$

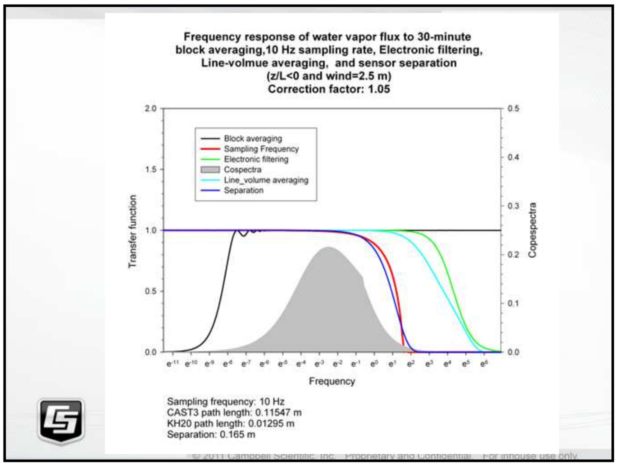
- A longer averaging period gives more low-frequency response.
- Must avoid diurnal drift.

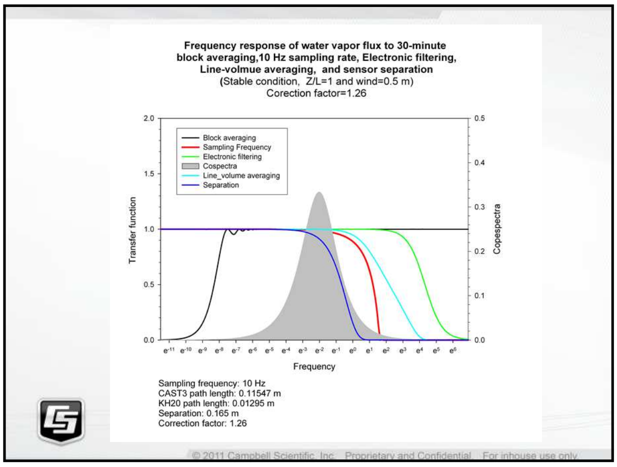
Typical averaging periods 30 mins

may be too short to capture LF covariance

- LF covariance at Manaus tropical forest lost for averaging periods < ~3 h. Know thy site!




Frequency response of water vapor flux to 30-minute
block averaging,10 Hz sampling rate, Electronic filtering,
Line-volmue averaging, and sensor separation
(z/L<0 and wind=2.5 m)
Correction factor: 1.05
Block averaging
Sampling Frequency
Electronic filtering
Cospectra
Line_volume averaging
Separation
Transfer function
Copespectra
Frequency
Sampling frequency: 10 Hz
CAST3 path length: 0.11547 m
KH20 path length: 0.01295 m
Separation: 0.165 m


Frequency response of water vapor flux to 30-minute
block averaging,10 Hz sampling rate, Electronic filtering,
Line-volmue averaging, and sensor separation
(Stable condition, Z/L=1 and wind=0.5 m)
Corection factor=1.26
Block averaging
Sampling Frequency
Electronic filtering
Cospectra
Line_volume averaging
Separation
Transfer function
Copespectra
Frequency
Sampling frequency: 10 Hz
CAST3 path length: 0.11547 m
KH20 path length: 0.01295 m
Separation: 0.165 m
Correction factor: 1.26
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Questions ?


谢谢!
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